Integrating Mathematical Modeling Thoughts into Probability and Mathematical Statistics Course Teaching

Xiaobo Liu

School of Information Engineering, Nanjing Xiaozhuang University, Nanjing, China

Abstract: Integrating thought method, teaching method and teaching means of mathematical modeling into Probability and Mathematical Statistics course teaching, and connecting Abstract concepts with specific life events could help students understand mathematics, enhance their learning initiative and cultivate their application awareness.

1. Introduction

Mathematical modeling is a means to establish a mathematical model through quantitative analysis of practical problems and solve problems with mathematical knowledge. College mathematics is more theoretical and Abstract, and many college freshmen habitually apply the thinking mode of middle school when they are first exposed to college mathematics, so they feel helpless in learning. Integrating mathematical modeling thought into university mathematics teaching, and connecting the Abstract concepts with specific life events could help students understand and memorize math concepts, could arouse the students' interest in learning, enhance the initiative of learning, also can let students realize the importance of modeling in our work and life, initially formed mathematical modeling thought, which would do good to their future work and study.

Through the practice of mathematical modeling, we realize that mathematical theory is closely related to the application of engineering technology, and the bridge between them is mathematical modeling. Mathematical modeling is an important means to cultivate students' innovative thinking, consciousness and ability.

2. The thinking method of integrating mathematical modeling into college mathematics courses

Mathematical modeling is not only the intersection of engineering technology, management science, economics and mathematics, but also the intersection of various mathematical branches. The establishment of mathematical models from practical problems may involve various branches of mathematics. Calculus is widely and deeply applied in various fields. Matrix contract or similar diagonalization in linear algebra is not only a pure mathematical operation, but also widely used in biology, economics, electronic engineering and other fields. Each basic course of university mathematics has its own characteristics. Probability and mathematical statistics belongs to the category of applied mathematics, which is a mathematical subject studying the statistical regularity of random phenomena. One of its characteristics is that it has a strong engineering and practical background, which is similar to the course of mathematical modeling. Students need to have a strong ability to analyze practical problems and extract mathematical models.

In China, more students are accustomed to learning in the form of "teacher speaking, memorizing by themselves and memorizing hard", lacking the ability to think independently and ask questions. The process of mathematical modeling is a process of creation and discovery. A practical problem can be observed, analyzed and discussed from various perspectives. Different mathematical methods and ideas can be adopted to establish completely different mathematical models. The teaching of mathematical modeling should pay attention to the cultivation of students' creative thinking and innovative consciousness, put practice in an important position, and aim at

improving students' ability to engage in modern scientific research and engineering technology development. It is a way worth exploring and practicing to update the teaching guiding ideology in the basic mathematics course and to integrate the teaching methods and means into mathematical modeling course. Teachers should try to change students' passive study way in the teaching, take student-centered discussion, heuristic and imitate teaching way, to show students the thinking of applied mathematics, guides the student analyze, think and ask question more, encourage students connect the basic knowledge of mathematics with the real world, and with the actual application problem, gradually forme their own modeling capabilities.

3. Examples of integrating mathematical modeling thoughts and methods into Probability and Mathematical Statistics course teaching

Here we will introduce in detail our thinking and practice of integrating the thoughts and methods of mathematical modeling into Probability and Mathematical Statistics course.

Blending is not the same as simply plugging, that is inserting mathematical modeling examples into the textbook, or explaining one or two mathematical model examples. In this way, although it can stimulate students' interest in applied mathematics to a certain extent, it is far from cultivating students' ability to build mathematical models by themselves. On the basis of doing not affect the curriculum system of Probability and Mathematical Statistics course, it should be fully integrated with the curriculum knowledge to achieve real integration.

The definitions, concepts and theorems of Probability and Mathematical Statistics have a strong engineering background and need to master more random models. Using Probability and Mathematical Statistics knowledge to solve practical problems is similar to the mathematical modeling process, which requires sufficient analysis, putting forward hypotheses, introducing variables, establishing coordinates, establishing one's own random mathematical model, seeking its mathematical solution, and giving the solution to practical problems. Finally, the whole process needs to be expressed in clear language. The learning difficulty of students lies in their inability to extract mathematical models from actual random phenomena. Teachers should strive to combine theory with practice through classroom teaching to analyze problems and guide students to analyze, think and ask more questions.

3.1 Introduction of Probability concept

Probability is the most basic concept, which involves the probability calculation of specific random events in middle school. In college, if the axiomatic definition of probability is given directly, students will have difficulty in understanding the concepts such as sample space and sample point, which are disjointed from the practical application. We take the way of interacting with students, let students participate in the process from actual to Abstract, gives a large number of random sample tests describing random phenomenon, let the student list test random events, analyze the relationship and structure of the listed events under the guidance of teachers, find out the special event, basic events, and make students realize the most can use basic events to express, and introduce the combination or operation thought of events. On the basis of introducing the definition of sum, plus and multiplication of events in words, the question is raised: what kind of operation form can be used to describe the operations and relations between events? At this time, the teacher shows the mathematical modeling thinking process that captures the essence of mathematics to the students, guides the students to grasp the essential characteristics of the basic events from the text description that is easy to understand, and uses mathematical symbols to express them. In this way, the sample points and sample spaces are naturally introduced, and the transformation process from events to sets is completed. Through process of extracting mathematical structure, students can establish the Abstraction and description of random events, which is an opportunity for students to experience and imitate Abstract mathematics from practice, deepen their understanding of what they have learned, and gradually form their own ability.

3.2 Geometric scheme problem

A peoson and B person meet in the period of 0 to T and in the scheduled place, the first person would wait for another person, and he would leave after the time t(t<T), solve the possibility that two people could meet.

This is a typical meeting problem, it should first establish a mathematical model and analyze it to solve this problem:

(1) Setting that it is equally possible for each person to arrive at the place and at each time between 0 and T;

(2) The times what two persons arrive are not have connection in each other.

Setting x and y represent the arrival time of A person and B person respectively, then $0 \leq x \leq T, 0 \leq y \leq T$,

To be able to meet, $|x-y| \leq t$, then we establish a rectangular coordinate plane, draw the area where two people may meet in the coordinate plane, this is a geometric probability model, then use geometric probability to calculate the probability, we know the probability of two people meet.

3.3 Introduction of random variables and distribution functions

The introduction of random variables is a leap in the development of probability theory. For students, it is also an important turning point from the probability calculation of individual events in middle school to the overall description of random problems. Students have mastered the concept of random variables and their distribution functions from a theoretical perspective, and can do some traditional exercises, which is far from solving practical random problems with what they have learned.

When introducing the five classical distributions: binomial distribution, poisson distribution, exponential distribution, uniform distribution and normal distribution, I often raises the following questions to students:

What are the experimental prototypes for these distributions?

How do you determine which distribution of the actual random variable obeys?

Are these distributions intrinsically related?

Why is a normal distribution so widespread in the real world?

We should lead the students to think and leave a guess to left foreshore for introduction of the center limit theorem, Poisson's theorem and other content.

For example, when introducing uniform distribution, after focusing on making students understand uniformity through examples, we should let them put forward the random variable subject to uniform distribution that they consider, and discuss in class, students have a full understanding of the random variable described by uniform distribution. We often use large random questions throughout the entire teaching process for students to discuss in class. In class, students use deductive thinking, analogical thinking, inductive thinking and other ways to analyze and draw their own conclusions from the actual background of mathematical theory or model under the teacher's guidance, and then compare with the teacher's conclusion or the conclusion in the textbook, which often makes students get unexpected surprises.

3.4 Full probability formula

Autosomal genetic model. The teaching process is roughly as follows :(1) practical problems. In autosomal inheritance, the offspring inherit one gene from each parent pair and form their own genetic pair, also known as genotype. The genotypes of some plants in botanical garden are AA, Aa and aa. How will the distribution of the three genotypes in the n generation of this plant change after several years? Can the breed be purified in this way? (2) model building. Guide students to use the full probability formula to establish the recursive relationship between the distribution of three genotypes in the n generation and the distribution of the n-1 generation. (3) model analysis and evaluation. The result of limit is used to explain the scientificity of this method.

3.5 Poisson theorem

In general, teaching poisson theorem is the proof of the theorem, and then elaborating poisson distribution can be regarded as the limit distribution of binomial distribution. However, how to judge whether poisson distribution can be used to describe an actual random variable is still unclear to students. After proving the theorem, we guide the students to analyze the conditions of the theorem $\lim\limits_{n \to \infty} np_n = \lambda \, (\lambda > 0)$

It is concluded that the parameters of the binomial distribution random variables sequence column { p_n } order infinitesimal, with sequence { $\dfrac{1}{n}$ } is deduced when n is enough big, p is small, the available parameters for lambda λ=np poisson distribution make approximation to the binomial distribution probability counting, and at the same time to make the students recognize the n times repeated trials of independence rare event (small probability events) the number of occurrences of approximately obey the poisson distribution can be thought of, can be used as a guess the basis of actual random variable obeying poisson distribution, and tell the students part in statistical hypothesis testing is to guess test method. In the interactive process of the explanation of poisson theorem, students experienced an important method of mathematical modeling: hypothesis → proof (test) hypothesis.

3.6 Law of large numbers and Monte Carlo simulation

In the first chapter of probability theory, the stability of frequency is introduced, and the essential difference between probability and frequency is usually emphasized. When introducing the concept of random variable, students will understand that frequency is a random variable. We know about Bernoulli's Law of large Numbers

$$\lim_{n \to \infty} p\{\left|\frac{m}{n} - p\right| < \varepsilon\} = 1, \ \text{ for any } \varepsilon > 0$$

In addition to explaining the theorem in detail in theory to make students understand the theorem deeply, the paper also used the computer simulation of Puffon's needle throwing experiment in class, through the demonstration to make students vividly realize the stability of the frequency presented when the number of experiments n is very large, and set up the idea of frequency estimation probability. Through the introduction of Monte Carlo method, students are told that the deterministic problem can be solved approximately by stochastic simulation, such as using Monte Carlo method to estimate PI, approximate calculation of definite integral, approximate solution of differential equation, etc. This explanation makes students fully understand the vitality of the theorem, so they would produce their own ideas to try.

4. Conclusion

Integrating the thought method, teaching method and teaching means of mathematical modeling into college mathematics courses could cultivate students' interest in learning mathematics and enable them to better absorb and master this basic knowledge. It also can train the student's application consciousness, is a very good development direction of mathematics teaching. These are just a few examples from the probability course. How to successfully integrate mathematical modeling ideas into the basic courses of mathematics in universities is worth further exploration.

References

[1] Jiang Qiyuan, Xie Jinxing, Ye Jun, Mathematical model (2nd edition), Higher Education Press, 2010.

[2] Bi Xiaohua, Xu Jun, Ye Jun, A preliminary study on the application of mathematical modeling to mathematics teaching in undergraduate course, Education and Vocation, vol.9, 2011.